SCIENCE FOR GLASS PRODUCTION

UDC 666.1

MATHEMATICAL SUPPORT FOR THE CAD TECHNOLOGICAL SUBSYSTEM OF A GLASS-MELTING FURNACE

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Translated from Steklo i Keramika, No. 7, pp. 3 – 8, July, 2004.

The advisability of constructing a CAD system for glass-melting furnaces is demonstrated. The main principles of the mathematical support for the CAD technological subsystem are formulated. The principles of constructing a mathematical model of the thermal performance of a furnace are described. The main equations providing for a combined solution of the exterior and the interior problems of heat and mass exchange taking into account the specifics of the physiochemical aspects of the glass-melting process are given.

The efficiency of investment projects in the glass industry to a great extent depends on the technical and economical parameters of the particular glass-melting furnace. The high capital cost of a modern furnace, which is primarily determined by the cost of refractory materials, can be justified in the case of high efficiency, low service cost, and a protracted furnace campaign [1]. The cost of constructing a furnace in Russia approaches these costs in industrial countries. Consequently, an investor has the right to expect a corresponding technical efficiency from the furnace. In this context the quality of the furnace design becomes critical.

It should be admitted that designing a furnace is traditionally reduced to the determination of geometrical sizes of particular furnace components. The design calculations at best include the determination of the thermal balance items for the melting and working zones of the furnace, the volume of the checkerwork regeneration, and other local calculations (parameters of the smoke funnel, burners, etc.). This scope of calculations can suffice only for a draft design of the furnace, which traces its layout, the positions of its fundamental supports, and the loads on them. The detail design of the furnace implies a deeper and more comprehensive account of all physicochemical aspects of the glass-melting process.

Regarding a glass-melting furnace as a continuous energy-technological melting aggregate, one can isolate two interrelated sets of processes that determine the design and efficiency of the furnace. The first set consists of generation of thermal energy in the working space and in the tank (additional electric heating); the second one implies the transfer of heat to the process zone. The thermal stress value and its distribution over the flame space of the furnace are determined by the fuel combustion intensity and gas dynamics of the combustion products flow, i.e., the set of the radiation and aerodynamic characteristics of the flame. Other terms being equal, the length of the flame, its velocity, and its position with respect to the heated surface have a determining effect on temperature distribution over the glass melt surface and the enclosing refractory brickwork. The above parameters finally determine the efficiency of exterior heat exchange and the integral heat absorption in the tank [2].

The distribution of the heat absorption of the melt in the volume of the process zone to a great extent depends on the thermophysical characteristics of the melt and the hydrodynamics of the tank. The intensity and nature of convection glass flows depend not only on the temperature field of the glass melt surface, but also on the design of the melting tank and the location of the bubbling system and barrier devices. When additional electric heating is used, the location and power of additional thermal power sources have a fundamental importance for the heat absorption and hydrodynamics of the melting tank.

Thus, at the stage of a detail design, which is the second design phase, it is necessary to obtain full quantitative information on the temperature fields of all furnace elements participating in heat exchange. This includes the temperature distribution on the roof and on the lateral walls of the upper furnace structure, the melt surface, the walls and the bottom of the melting tank, and the gaseous space of the furnace and the melt. This information assists in making a justified decision on the optimal characteristics of the flame, the choice of refractory materials for brickwork elements that undergo the

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most substantial wear, identifying stagnation zones in the melting tank, determination of the final layout of the barrier devices, and, if necessary, the input site and power of additional electric heating. Such information, as a rule, promotes a reconsideration of the initial geometric layout of the furnace.

Based on the above, it can be stated that a correct design of a glass-melting furnace can be implemented only applying an automated design system, of which CAD is the optimum form. The development of CAD for a glass-melting furnace is a most complicated problem. The first stage of solving this problem involves the development of a number of local subsystems to provide solutions for geometrical and technological design problems. The most complicated subsystem is the technological design subsystem, which cannot be created without corresponding technical, mathematical, software, and informational support.

Analysis indicates that the technical support of a CAD subsystem includes an up-to-date PC with an extended peripheral infrastructure. The majority of the design procedures, including the construction of functional models, geometrical design problems, and numerical methods for solving algebraic and differential equations are implemented using standard software products. Published results of theoretical and experimental studies of glass melting and the thermophysical properties of batches, melts and refractories provide a database that can be used as general informative support. Thus, the main difficulty in developing a technological design subsystem involves the development of its mathematical and special informational support based on a mathematical model of the thermal performance of a glass-melting furnace. Let us discuss the main concepts of this model.

Three groups of equations were used to formalize heat and mass exchange processes in a glass-melting furnace:

– exterior problem equations describing the regularities of heat release and mass transfer in fuel combustion, radiant and convective heat exchange between hot and burning combustion products and surfaces of the tank and enclosing brickwork of the working space;

– interior problem equations determining the propagation of heat received by the tank surface over the tank volume, taking into account the kinetics of physicochemical transformations and heat losses via the tank brickwork;

– tank hydrodynamics equations describing the effect of the parameters of the exterior and interior problems on the three-dimensional convection of the glass melt.

In developing the numerical model of the furnace, the main system theory requirement (rule) was taken into account, namely, that all processes taking place in a object constitute an integrated system and are interrelated and mutually determined [3].

The resolvent zonal method [3, 4] was used for solving the exterior problems. This method implies splitting the melting zone of the glass-melting furnace into volume and surface zones with preset geometrical sizes. The gas spaces is partitioned into volume zone, whereas the enclosing brickwork and the glass melt mirror are split into surface zones.

The heat exchange energy characteristics for each particular zone are determined by solving a system of nonlinear algebraic equations of thermal balance and heat transfer written in the following form:

$$\sum_{j=1}^{n+m} a_{ji} T_i^4 + \sum_{j=1}^{n+m} g_{ji} T_i + Q_i = 0, \quad i = 1, ..., m+n, \qquad (1)$$

where m and n are the quantities of the volume and surface zones, respectively; a_{ji} is the coefficient of radiation exchange between zones j and i; T_i is the temperature of zone i, K; g_{ij} is the coefficient of mass exchange and convective exchange between zones j and i; Q_i is a free summand of the equation taking into account the exterior and interior heat sources and outlets.

The radiation exchange coefficients characterizing the quantity of radiation energy transferred from zone j to zone i at $T_i = 1$ are found from the equations:

for volume zones

$$a_{ji} = 4V_j \sigma_0 k_j f_{ji};$$

for surface zones

$$a_{ji} = F_j \sigma_0 \varepsilon_j f_{ji},$$

where V_j and F_j are the volume and the surface area of the respective zones; σ_0 is the Stefan – Boltzmann constant, $\sigma_0 = 56.687 \times 10^{-12}$ kW/(m^2 · K); k_j is the reduction coefficient of zone j, m^{-1}; f_{ji} is the resolving generalized angle factor characterizing the share of energy absorbed by zone i from zone j taking into account that part of the energy of the radiation ray is absorbed by gas; ε_j is the degree of blackness of the surface zone j.

The coefficients f_{ji} are found by solving a system of linear algebraic equations using the algorithms proposed by V. G. Lisienko and Yu. A. Zhuravlev [5, 6]:

for volume zones

$$f_{ji} = \psi_{ji} + \sum_{k=1}^{n} r_k \psi_{jk} f_{ki}; \qquad (2)$$

for surface zones

$$f_{ji} = \psi_{ji} \varepsilon_i + \sum_{k=1}^{n} r_k \psi_{jk} f_{ki}, \quad i, j = 1, 2, ..., m+n, \qquad (3)$$

where ψ_{ji} is the generalized angle coefficient of the radiation from zone j to zone i; ψ_{jk} is the same for radiation transferred from a surface zone, for instance, k; $r_k = 1 - \varepsilon_k$ is the reflecting capacity of the zone k.

In Eqs. (2), (3) the first summand in the right-hand side characterizes the direct heat transfer from zone j to zone i. The second summand describes the heat transfer between

these zones taking into account the reflection of radiation from other surface zones k that are "visible" for zones j and i.

The coefficients ψ_{ji} are calculated in accordance with the statistical test method using the algorithm proposed by A. É. Klekl' and S. D. Dreizin-Dubchenko [7, 8]. Each fixed element i in this set corresponds to a system of $m + n$ linear equations. The ith column of the matrix ψ is found on the right-hand side of this system. The unknown quantities are the elements of the ith column of the matrix f. Thus, to calculate the resolving angle coefficients with the constraint $i = 1, \ldots m + n$, it is necessary to solve $m + n$ systems of linear equations that differ only in their right-hand sides. This is implemented using a standard subprogram in Compaq Visual Fortran 6.6.B Professional.

The summand Q_i depends on the specific statement of the problem and includes heat release in fuel combustion, the heat consumed to compensate the thermal effect of chemical reactions, the heat used for heating of the batch, cullet, and the glass melt, the heat losses via the walls and bottom of the furnace, etc.

The system of equations (1) has many parameters whose values are determined by iteration. This primarily concerns the radiation characteristics of combustion products, which depend on the temperature and composition of the gas. Therefore, a combined approach is used for modeling the thermal work of a glass-melting furnace. Initially the temperature of the combustion products and the degree of burn-out of fuel in particular zones of the working space are set using the typical conditions of the operation. According to the preset values of the degree of combustion, the composition of the gaseous phase is determined and the required characteristics are calculated using the standard method [9]. The procedure described is repeated in the context of the zonal method until the convergence of the results.

Nevertheless, the calculation results in such an approximation cannot be regarded as final, since the zonal method does not take into account the regularities of the gas flow and fuel combustion under this flow. To eliminate this drawback, the zonal-node method is used [10]. The temperature fields and the combustion product fields determined by the zonal method are approximated by continuous functions using the Table Curve 3D software package. The working space of the furnace is covered by a nonuniform finite-difference grid in such a way that the nodes of the volume (gaseous) zones coincide with the grid nodes. The flow and heat and mass transfer equations for the working space of the furnace are solved using the SIMPLER method developed by S. Patankar [11] using the approximation results as initial approximation.

The most general form of gas mechanics and heat and mass transfer equations taking into account the combustion of fuel is presented in [12, 13]. The mathematical model of the glass-melting furnace took into account the specifics of the thermal performance of the particular furnace related to the cross-link effects, i.e., the mutual influence of specific phenomena on each other. Thus, it was taken into account that the mass transfer caused by barodiffusion with respect to the working space of the glass-melting furnace can be neglected due to the low pressure gradient as compared to the pressure itself. Within the temperature interval up to 2500 K the values of the thermal diffusion coefficient are within the interval of $10^{-3} - 10^{-5}$. Therefore, although the temperature gradient at the combustion front may be substantial, the mechanism of thermal diffusion in general can be neglected as well. Thus, the equations of the "fuel" and "oxidizer" balance in the mathematical model of the glass-melting furnace have the following form:

$$\rho\left(\frac{\partial m_{\mathrm{f}}}{\partial \tau} + \vec{v}\,\mathrm{grad}\, m_{\mathrm{f}}\right) = \mathrm{div}\,(\rho D_{\mathrm{ef.}\,i}\,\mathrm{grad}\, m_{\mathrm{f}}) + R_{\mathrm{f}}; \qquad (4)$$

$$\rho\left(\frac{\partial m_{\mathrm{ox}}}{\partial \tau} + \vec{v}\,\mathrm{grad}\, m_{\mathrm{ox}}\right) = \mathrm{div}\,(\rho D_{\mathrm{ef.}\,i}\,\mathrm{grad}\, m_{\mathrm{ox}}) + R_{\mathrm{ox}}, \qquad (5)$$

where ρ is the density of the mixture of gas, air, and combustion products, kg/m^3; m_{f} and m_{ox} are the mass contents of the "fuel" and the "oxidizer"; $D_{\mathrm{ef.}i}$ is the effective diffusion coefficient of the ith component in the mixture, m^2/sec; R_{f} and R_{ox} are the flow rates of the "fuel" and the "oxidizer", kg/(sec $\cdot$ m^3); $\vec{v}$ is the mass center velocity vector with components $v_1 = v_x$, $v_2 = v_y$, $v_3 = v_z$, m/sec; z_i is the generalized coordinate with the components $x_1 = x$, $x_2 = y$, and $x_3 = z$ (i.e., $i, j = 1, 2, 3$), m.

For a simple chemical reaction where two components (the fuel and the oxidizer) react in a certain ratio [1 kg "fuel" + i kg "oxidizer" $\rightarrow$ $(1 + i)$ kg reaction products], their flow rates are found from the expressions

$$R_{\mathrm{f}} = Kp^2 m_{\mathrm{f}} m_{\mathrm{ox}} \exp\left[-\frac{E}{RT}\right];$$

$$R_{\mathrm{ox}} = i R_{\mathrm{f}}, \qquad (6),$$

where K, E are empirical coefficients determined by the kind of fuel and by the type of its combustion.

The closure of the gas mass balance for the working space is implemented taking into account the equation of the total gas (mixture) continuity:

$$\frac{\partial \rho}{\partial \tau} + \mathrm{div}\,(\rho \vec{v}) = 0. \qquad (7)$$

Using the mass center velocities $\vec{v}$, the equations of compressible gas flow with variable viscosity have the form of the traditional Navier equations:

$$\rho \frac{\mathrm{d}\vec{v}}{\mathrm{d}\tau} = -\mathrm{grad}\, p + \mathrm{Div}\left[\mu_{\mathrm{ef}}\left(\frac{\partial \nu_i}{\partial z_j} + \frac{\partial \nu_j}{\partial z_i}\right) - \frac{2}{3}\mu_{\mathrm{ef}}(\mathrm{div}\,\vec{v})\delta\right] + \sum_{k=1}^{2} \rho_k g, \qquad (8)$$

where ρ_k is the partial density of the kth component of the mixture of gases, kg/m^3; τ is the time, sec; p is the mixture

pressure, Pa; μ_{ef} is the effective dynamic viscosity of the mixture, Pa · sec; δ is the identity second-rank tensor, whose elements are Kroneker symbols δ_{ij}; g is the acceleration vector of the gravity per mass unit of the component k; Div is the stress tensor divergence; div is the vector divergence; $d\vec{v}/d\tau$ is the total or substantial velocity derivative (acceleration) of the mixture determined by the relation

$$\frac{d\vec{v}}{d\tau} = \frac{\partial \vec{v}}{\partial \tau} + (\vec{v}\,\nabla)\,\vec{v},$$

where ∇ is the vector differential operator "nabla" [3].

The gas flow in the working space of the glass-melting furnace has a clearly expressed turbulent nature, hence, the effective coefficients $\mu_{ef} = \mu + \mu_t$ and D_{ef} depend on the degree of the flow turbulence. There are several engineering theories of turbulence known. The most convenient in the practical aspect is the Kolmogorov – Prandtl model $E_t - \varepsilon$ [3, 14], which has been used in our study. According to this model, the turbulent component of the effective dynamic viscosity μ_t is proportional to the square of kinetic turbulence energy E_t and inversely proportional to the velocity of the turbulence energy dissipation ε. Then the kinematic turbulence viscosity ν_t is determined by the expression

$$\nu_t = \frac{\mu_t}{\rho} = \frac{C_\mu E_t^2}{\varepsilon}, \qquad (9)$$

where C_μ is the empirical coefficient; $C_\mu = 0.09$; E_t is the kinetic turbulence energy per weight unit:

$$E_t = 0.5\,[\langle(\nu_x')^2\rangle + \langle(\nu_y')^2\rangle + \langle(\nu_z')^2\rangle];$$

here the angular brackets denote the time averaging and the primes indicate the pulsation constituents (fluctuations) of the velocity components.

Specific balance equations (transfer equation) are implemented for the variables E_t and ε.

The equation of kinetic turbulence energy transfer is

$$\frac{\partial E_t}{\partial \tau} + \vec{v}\,\mathrm{grad}\,E_t = \mathrm{div}\left[\left(\nu + \frac{\nu_t}{\mathrm{Pr}_t^{\nu}}\right)\mathrm{grad}\,E_t\right] + G - \varepsilon, \qquad (10)$$

where $\nu = \mu/\rho$ is the molecular kinetic viscosity; G is the turbulence generation.

For the calculation of G we use the expression

$$G = \nu_t\left[2\left(\frac{\partial \nu_x}{\partial x}\right)^2 + \left(\frac{\partial \nu_x}{\partial y} + \frac{\partial \nu_y}{\partial x}\right)^2 + 2\left(\frac{\partial \nu_y}{\partial y}\right)^2 + \left(\frac{\partial \nu_x}{\partial z} + \frac{\partial \nu_z}{\partial x}\right)^2 + \left(\frac{\partial \nu_y}{\partial z} + \frac{\partial \nu_z}{\partial y}\right)^2 + 2\left(\frac{\partial \nu_z}{\partial z}\right)^2\right]. \qquad (11)$$

The equation of the transfer of turbulence energy dissipation velocity is

$$\frac{\partial \varepsilon}{\partial \tau} + \vec{v}\,\mathrm{grad}\,\varepsilon = \mathrm{div}\left[\left(\nu + \frac{\nu_t}{\mathrm{Pr}_t^{\nu}}\right)\mathrm{grad}\,\varepsilon\right] + C_1\frac{\varepsilon G}{E_t} - C_2\frac{\varepsilon^2}{E_t}, \qquad (12)$$

where C_1, C_2, Pr_t^{ν} and Pr_ε are empirical coefficients taken from the intervals $\mathrm{Pr}_t^{\nu} = 0.9 - 1.0$, $\mathrm{Pr}_\varepsilon = 1.22 - 1.3$, $C_1 = 1.44 - 1.55$, and $C_2 = 1.92 - 2.0$.

The gas flow energy equation (the Fourier – Kirchhoff heat transfer equation) takes into account all the main and accessory (cross-link) effects of its enthalpy variation [3, 12]. The summands on the right-hand side of the equation characterize heat transfer by radiation, heat conduction, and diffusion heat conduction, as well as heat transfer due to the pressure gradient; heat sources (outlets) caused by chemical reactions; work of pressure forces; heat sources cause by flow energy dissipation, i.e., by the conversion of the internal friction work into heat; heat transfer due to the diffusion mass flow and heats sources caused by the work of exterior forces in the diffusion mass flow process.

Some enthalpy variation sources, as applied to the conditions of the glass-melting furnace, can be neglected in view of their small values. It has been noted that the pressure gradient is very small; consequently, the thermal effect of pressure can be neglected. The work of pressure forces and aerodynamic heating is not significant as well. Heat transfer under the effect of gravity and the diffusion thermal effect are extremely insignificant. Consideration of the above circumstances allows for a perceptible simplification of the equation of the energy of the medium. At the same time, additional aspects should be taken into account. This primarily concerns the thermal conductivity of the glass melt, which should be replaced by its effective value $\lambda_{ef.gm}$ equal to the sum of its conductive and radiation components. Next, one should take into account the value E_t. Furthermore, the Fourier – Kirchhoff equation should be transformed for turbulent flow conditions.

Combining the energy equation with the equation of the gas mixture continuity (7), we can write the equation of the thermal balance of gases suitable for the analysis of heat transfer in the working space of the glass-melting furnace:

$$c_p\left[\frac{\partial}{\partial \tau}(\rho T) + \mathrm{div}(\rho \vec{v} T)\right] = \mathrm{div}\,(\lambda_{ef}\,\mathrm{grad}\,T + q_R) - \sum_{j=f}^{ox} H_j R_j + \mathrm{div}\left[\mu_{ef}\,\mathrm{grad}\,\frac{\vec{v}^2}{2} + \Gamma_k\,\mathrm{grad}\,E_t + \rho\sum_{j=f}^{ox} Q_j^* D_{ef.j}\,\mathrm{grad}\,m_j\right] - \sum_{j=f}^{ox} c_{pj} J_j^*\,\mathrm{grad}\,T, \qquad (13)$$

where T is the gas temperature, K; c_p is the specific heat capacity of the gas at constant pressure, J/(kg · K); q_R is the ra-

diation flux density, W/m^2; λ_{ef} is the thermal conductivity of gases, W/(m · K); Q_j^* is the heat transfer of the *j*th component of the gas mixture (the heat transferred by one mole of this component as a consequence of diffusion at grad $T = 0$), J/kg; H_j is the thermal effect from the reaction of the consumption of the *j*th component, J/kg; J_j^* is the diffusion flux density, kg/(m^2 · sec); $\Gamma_k = \nu + \nu_t/\mathrm{Pr}_t^{\nu}$ is the transfer (diffusion) coefficient of turbulence energy, m^2/sec.

Note that Eq. (13) is integrodifferential, since the vector of the radiation flux density q_R has the following components:

$$q_{Ri} = \int_{4\pi} In_i \, d\Omega ,$$

where I is the radiation flux intensity; n_i is the direction cosine of the thermal ray with respect to the *i*th axis of the coordinate system; $d\Omega$ is the elementary body angle.

The system of equations (4) – (13) is solved under the boundary conditions of the 3rd kinds on the walls and the roof of the furnaces and under the boundary conditions of the 4th kinds (conjugacy conditions) on the surface of the melting tank. The dynamics of fuel combustion is calculated in accordance with Eqs. (4) – (7) in the context of the Schwab – Zel'dovich approximation [11].

The results of cyclic solving of zonal method equations (1) – (3) and gas dynamics and heat and mass transfer equations (4) – (13) are consistent fields of gas temperatures in the working space of the furnace and on the enclosing surfaces, gas composition, velocities, and pressures. This makes it possible to relate the zonal method to the regularities of gas flow, fuel combustion, and heat exchange in the working space volume, approach the problem of optimizing the design and regime parameters of the fuel-burning devices (burner nozzle diameters, layout of the burners, selection of the angle of the burner with respect to the tank surface, etc.), and finally optimizing the geometrical parameters of the furnace.

It should be noted that the above cited exterior problem equations form an interrelated mutually determined chain. In principle they characterize the energy aspects of the glass-melting process. In this sense the concluding equations of the mathematical model, i.e., the equations of the tank hydrodynamics, have a certain independence. Their solution requires only information on heat flux densities on the glass melt surface and temperatures on this surface from the exterior problem. Although these equations are equations of convective heat exchange between the melt and the enclosing surfaces, i.e., they describe thermal energy transfer regularities, the purpose of this block of the mathematical model is purely technological. It is intended to identify the thermal conditions of the furnace providing for an optimum homogeneity of the glass melt.

The tank hydrodynamics equations include:
glass melt flow equations

$$\rho_{gm}\left[\frac{d\vec{v}_{gm}}{d\tau} + (\vec{v}_{gm}\nabla)\vec{v}_{gm}\right] = - \operatorname{grad} p_{gm} + \operatorname{Div}(\mu_{gm} \operatorname{grad} \vec{v}_{gm}) + \rho_{gm} g; \quad (14)$$

the equation of glass melt continuity

$$\frac{\partial \rho_{gm}}{\partial \tau} + \operatorname{div}(\rho_{gm} \vec{v}_{gm}) = 0; \quad (15)$$

the equation of glass melt energy combined with the continuity equation

$$c_{gm}\left[\frac{\partial}{\partial \tau}(\rho_{gm} T_{gm}) + \operatorname{div}(\rho_{gm} \vec{v}_{gm} T_{gm})\right] = \operatorname{div}(\lambda_{ef.gm} \operatorname{grad} T_{gm}), \quad (16)$$

where ρ_{gm} is the glass melt density, kg/m^3; $\vec{v}_{gm}$ is the glass melt flow velocity, m/sec; p_{gm} is the glass melt pressure, Pa; μ_{gm} is the glass melt dynamic viscosity, Pa · sec; c_{gm} is the specific heat capacity of the glass melt, J/(kg · K); T_{gm} is the glass melt temperature, K.

The equations of the discrete analog of system (14) – (16) in the three-dimensional version are solved by the SIMPLER method; two-dimensional approximation is implemented in the "vorticity – current function" variables [3, 14]. Since the components of the glass melt velocity on a free surface (open channel) are required in any case, the "creep flow" problem of glass melt in the tank is primarily solved. Note that the reliability of solving the tank hydrodynamics equations to a great extent depends on the reliability of data on the thermophysical properties of the glass melt and the dynamics of physiochemical processes occurring in its formation and melting.

To conclude, it should be noted that the mathematical model of the thermal performance of a glass-melting furnace considered here enables one to study the regularities of the exterior and interior heat and mass transfer. The analysis of these regularities will make it possible to solve a number of problems in the optimization of different functional relationships and thus to create a special informational support for the subsystem of the technological design of glass-melting furnaces.

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